11/27/2023 homework

$$1.(1)\int x^2(3-x^2)^2dx=\int x^2(x^4-6x^2+9)dx=\int(x^6-6x^4+9x^2)dx=\frac{1}{7}x^7-\frac{6}{5}x^5+3x^3+C$$

$$1.(5)\int\frac{\sqrt{x^3+x^{-3}+2}}{x^3}dx=\int\frac{\left|x^{\frac{3}{2}}+x^{-\frac{3}{2}}\right|}{x^3}dx=\int\frac{x^{\frac{3}{2}}+x^{-\frac{3}{2}}}{x^3}dx=\int\left(x^{-\frac{3}{2}}+x^{-\frac{9}{2}}\right)dx=-2x^{-\frac{1}{2}}-\frac{2}{7}x^{-\frac{7}{2}}+C$$

$$1.(8)\int\sqrt{1+\sin 2x}\,dx=\int|\sin x+\cos x|dx=\int\left|\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\right|dx=\sqrt{2}\int\left|\sin\left(x+\frac{\pi}{4}\right)\right|dx$$

$$=\sqrt{2}\left|\cos\left(x+\frac{\pi}{4}\right)\right|+C$$

$$1.(10)\int\csc^2 2x dx=\frac{1}{2}\int\csc^2 2x d2x=\frac{1}{2}\int\csc^2 2x d2x=\frac{1}{2}\int d(-\cot 2x)=-\frac{1}{2}\cot 2x+C$$

$$1.(12)\int\frac{dx}{x^4(1+x^2)}=\int\frac{dx}{x^4}-\int\frac{dx}{x^2(1+x^2)}=\int\frac{dx}{x^4}-\int\frac{dx}{x^2}+\int\frac{dx}{x^2+1}=-\frac{1}{3x^3}+\frac{1}{x}+\arctan x+C$$

$$2.(2)\int x\sqrt{2x-1}\,dx=\int\left(\frac{2x-1}{2}+\frac{1}{2}\right)\sqrt{2x-1}\,d\left(\frac{2x-1}{2}\right)=\frac{1}{4}\int(2x-1+1)\sqrt{2x-1}\,d(2x-1)$$

$$=\frac{1}{4}\int\left(\left(\sqrt{2x-1}\right)^3+\sqrt{2x-1}\right)d(2x-1)=\frac{1}{4}\int(2x-1)^{\frac{3}{2}}d(2x-1)+\frac{1}{4}\int(2x-1)^{\frac{1}{2}}d(2x-1)$$

$$=\frac{1}{4}\frac{2}{5}\int d(2x-1)^{\frac{5}{2}}+\frac{1}{4}\frac{2}{3}\int d(2x-1)^{\frac{3}{2}}=\frac{1}{10}(2x-1)^{\frac{5}{2}}+\frac{1}{6}(2x-1)^{\frac{3}{2}}+C$$

$$2.(3)\int\frac{x^2+1}{x^2-1}dx=\int\left(1+\frac{2}{x^2-1}\right)dx=\int dx+\int\frac{2}{x^2-1}dx=\int dx+\int\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx$$

$$=\int dx+\int\frac{1}{x-1}dx-\int\frac{1}{x+1}dx=x+\ln\left|\frac{x-1}{x+1}\right|+C$$

$$2.(5)\int\frac{dx}{x^2+2x-3}=\int\frac{dx}{(x+1)^2-4}=\frac{1}{4}\int\frac{dx}{\left(\frac{x+1}{2}\right)^2-1}=\frac{1}{4}\ln\left|\frac{\frac{x+1}{2}-1}{\frac{x+1}{2}+1}\right|+C$$

$$2.(6)\int\frac{x^3}{(x+1)\sqrt[3]{x+1}}dx=\int\frac{((x+1)-1)^3}{(x+1)\sqrt[3]{x+1}}d(x+1)=\int\frac{(x+1)^3-3(x+1)^2+3(x+1)-1}{(x+1)^{\frac{4}{3}}}d(x+1)$$

$$=\int\left((x+1)^{\frac{5}{3}}-3(x+1)^{\frac{2}{3}}+3(x+1)^{-\frac{1}{3}}-(x+1)^{-\frac{4}{3}}\right)d(x+1)$$

$$=\int\left(\frac{3}{8}d(x+1)^{\frac{8}{3}}-\frac{9}{5}d(x+1)^{\frac{5}{3}}+\frac{9}{2}d(x+1)^{\frac{2}{3}}+3d(x+1)^{-\frac{1}{3}}\right)$$

$$=\frac{3}{8}(x+1)^{\frac{8}{3}}-\frac{9}{5}(x+1)^{\frac{5}{3}}+\frac{9}{2}(x+1)^{\frac{2}{3}}+3(x+1)^{-\frac{1}{3}}+C$$

$$2.(8)\int\frac{dx}{\sqrt{x^2-1}\left(\sqrt{x+1}+\sqrt{x-1}\right)}=\int\frac{\sqrt{x+1}-\sqrt{x-1}}{2\sqrt{x^2-1}}dx=\frac{1}{2}\int\left(\frac{1}{\sqrt{x-1}}-\frac{1}{\sqrt{x+1}}\right)dx$$

$$=\frac{1}{2}\int d\left(2\sqrt{x-1}\right)-d\left(2\sqrt{x+1}\right)=\sqrt{x-1}-\sqrt{x+1}+C.$$

$$2.(10)\int\frac{1}{1-\sin 2x}dx=\int\frac{1}{(\sin x-\cos x)^2}dx=\int\frac{1}{2\sin^2\left(x-\frac{\pi}{4}\right)}dx=\frac{1}{2}\int d\left(-\cot\left(x-\frac{\pi}{4}\right)\right)=-\frac{1}{2}\cot\left(x-\frac{\pi}{4}\right)+C$$

6.2

$$1.(2)\int\frac{dx}{3+2x^2}=\frac{1}{3}\int\frac{dx}{1+\frac{2}{3}x^2}\overset{t=\sqrt{\frac{2}{3}}x}{=}\frac{1}{3}\sqrt{\frac{3}{2}}\int\frac{dt}{1+t^2}=\frac{1}{\sqrt{6}}\arctan t+C=\frac{1}{\sqrt{6}}\arctan\left(\sqrt{\frac{2}{3}}x\right)+C$$

$$1.(3)\int\frac{dx}{\sqrt{3-2x^2}}=\frac{1}{\sqrt{3}}\int\frac{dx}{\sqrt{1-\frac{2}{3}x^2}}\overset{t=\sqrt{\frac{2}{3}}x}{=}\frac{1}{\sqrt{3}}\sqrt{\frac{3}{2}}\int\frac{dt}{\sqrt{1-t^2}}=\frac{1}{\sqrt{2}}\arcsin t+C=\frac{1}{\sqrt{2}}\arcsin\left(\sqrt{\frac{2}{3}}x\right)+C$$

$$1.(5)\int\frac{xdx}{3+2x^2}=\frac{1}{2}\int\frac{dx^2}{3+2x^2}=\frac{1}{4}\int\frac{d(3+2x^2)}{3+2x^2}=\frac{1}{4}\ln(3+2x^2)+C$$

$$1.(6)\int\frac{xdx}{1+x^4}=\frac{1}{2}\int\frac{dx^2}{1+x^4}=\frac{1}{2}\arctan(x^2)+C$$

$$1.(7)\int\frac{dx}{\sqrt{x}(1+x)}=2\int\frac{d\left(\sqrt{x}\right)}{1+x}=2\arctan\left(\sqrt{x}\right)+C$$

$$1.(8)\int\frac{dx}{x^2e^{\frac{1}{x}}}=-\int\frac{d\left(\frac{1}{x}\right)}{e^{\frac{1}{x}}}=\int e^{-\frac{1}{x}}d\left(-\frac{1}{x}\right)=e^{-\frac{1}{x}}+C$$

$$1.(10)\int\frac{dx}{(x^2+1)^{\frac{3}{2}}}\overset{x=\tan t}{=}\int\frac{d\tan t}{(\tan^2t+1)^{\frac{3}{2}}}=\int\frac{\sec^2tdt}{\sec^3t}=\int\frac{dt}{\sec t}=\int\cos tdt=\sin t+C$$

$$=\sin(\arctan x)+C=\frac{x}{\sqrt{1+x^2}}+C$$

$$1.(12)\int\frac{xdx}{\sqrt{1+x^2}}=\frac{1}{2}\int\frac{d(1+x^2)}{\sqrt{1+x^2}}=\frac{1}{4}\int d\sqrt{1+x^2}=\frac{1}{4}\sqrt{1+x^2}+C$$

$$2.(1)\int x^2e^{-2x^3}dx=\frac{1}{3}\int e^{-2x^3}dx^3=-\frac{1}{6}\int e^{-2x^3}d(-2x^3)=-\frac{1}{6}e^{-2x^3}+C$$

$$2.(5)\int\cos^5x\sin^3xdx=-\int\cos^5x\sin^2xd\cos x=-\int(\cos^5x-\cos^7x)d\cos x=-\frac{\cos^6x}{6}+\frac{\cos^8x}{8}+C$$

$$2.(8)\int\frac{\sin x+\cos x}{\sqrt{\sin x-\cos x}}dx=\int\frac{1}{\sqrt{\sin x-\cos x}}d(\sin x-\cos x)=2\sqrt{\sin x-\cos x}+C$$

$$2.(11)\int\frac{dx}{1+\cos x}=\int\frac{dx}{2\cos^2\frac{x}{2}}=\int\frac{d\left(\frac{x}{2}\right)}{\cos^2\frac{x}{2}}=\tan\frac{x}{2}+C$$

$$3.(4)\int\frac{xdx}{2+3x^2+x^4}=\frac{1}{2}\int\frac{dx^2}{2+3x^2+x^4}=\frac{1}{2}\int\frac{dx^2}{\left(x^2+\frac{3}{2}\right)^2-\frac{1}{4}}=\frac{1}{2}\left(\int\frac{dx^2}{(x^2+1)(x^2+2)}\right)$$

$$=\frac{1}{2}\left(\int\frac{dx^2}{x^2+1}-\int\frac{dx^2}{x^2+2}\right)=\frac{1}{2}\ln\left(\frac{x^2+1}{x^2+2}\right)+C$$

$$3.(6)\int\frac{x^3dx}{\sqrt{1+x^2}}=\frac{1}{2}\int\frac{x^2dx^2}{\sqrt{1+x^2}}=\frac{1}{2}\int\frac{x^2+1-1}{\sqrt{1+x^2}}dx^2=\frac{1}{2}\int\sqrt{1+x^2}\,dx^2-\frac{1}{2}\int\frac{1}{\sqrt{1+x^2}}dx^2$$

$$=\frac{1}{3}(1+x^2)^{\frac{3}{2}}-(1+x^2)^{\frac{1}{2}}+C$$

$$3.(7)\int\frac{dx}{1+e^x}=\int\frac{de^x}{(1+e^x)e^x}=\int\frac{de^x}{e^x}-\int\frac{de^x}{1+e^x}=x-\ln(1+e^x)+C$$

$$3.(10)\int\frac{dx}{\sqrt{1+e^x}}=\int\frac{de^x}{e^x\sqrt{1+e^x}}\overset{t=\sqrt{1+e^x}}{=}\int\frac{d(t^2-1)}{(t^2-1)t}=\int\frac{2tdt}{(t^2-1)t}=\int\frac{2dt}{(t-1)(t+1)}$$

$$=\int\frac{dt}{t-1}-\int\frac{dt}{t+1}=\ln\frac{t-1}{t+1}+C=\ln\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}+C$$

$$3.(12)\int\frac{\arctan\sqrt{x}\,dx}{\sqrt{x}(1+x)}=2\int\frac{\arctan\sqrt{x}\,d\sqrt{x}}{(1+x)}=2\int\frac{\arctan\sqrt{x}\,d\sqrt{x}}{\left(1+\left(\sqrt{x}\right)^2\right)}=2\int\arctan\sqrt{x}\,d\arctan\sqrt{x}=\left(\arctan\sqrt{x}\right)^2$$

$$4.(6)\,p=x,q=(a^2-x^2)^{\frac{1}{2}}\Longrightarrow p^2+q^2=a^2\Longrightarrow pdp+qdq=0$$

$$\int\frac{dx}{(a^2-x^2)^{\frac{3}{2}}}=\int\frac{dp}{q^3}=-\int\frac{dq}{pq^2}=-\frac{1}{a^2}\int\frac{(p^2+q^2)dq}{pq^2}=-\frac{1}{a^2}\left(\int\frac{p}{q^2}dq+\int\frac{dq}{p}\right)$$

$$=-\frac{1}{a^2}\left(-\int pd\frac{1}{q}+\int\frac{dq}{p}\right)=-\frac{1}{a^2}\left(-\frac{p}{q}+\int\frac{dp}{q}+\int\frac{dq}{p}\right)=-\frac{1}{a^2}\left(-\frac{p}{q}+\int\frac{pdp+qdq}{pq}\right)$$

$$=\frac{1}{a^2}\frac{p}{q}=\frac{1}{a^2}\frac{x}{(a^2-x^2)^{\frac{1}{2}}}$$

$$4.(9)\,p=x,q=(a^2+x^2)^{\frac{1}{2}}\Longrightarrow q^2-p^2=a^2\Longrightarrow qdq-pdp=0$$

$$\Longrightarrow d(\ln(p+q))=\frac{d(p+q)}{p+q}=\frac{dq}{p}=\frac{dp}{q}=\frac{pdq}{p^2}=\frac{qdp}{q^2}=\frac{qdp-pdq}{q^2-p^2}=\frac{qdp-pdq}{a^2}$$

$$\int\frac{x^2dx}{\sqrt{a^2+x^2}}=\int\frac{p^2dp}{q}=\int\frac{p^2dq}{p}=\int pdq=\frac{1}{2}\left(\int pdq+\int qdp+\int pdq-\int qdp\right)$$

$$=\frac{1}{2}\left(\int d(pq)-a^2\int\frac{qdp-pdq}{a^2}\right)=\frac{1}{2}\left(\int d(pq)-a^2\int d(\ln(p+q))\right)=\frac{1}{2}(pq-a^2\ln(p+q))+C$$

$$=\frac{1}{2}\left(x(a^2+x^2)^{\frac{1}{2}}-a^2\ln\left(x+(a^2+x^2)^{\frac{1}{2}}\right)\right)+C$$

$$4.(12)\,p=x,q=\sqrt{x^2-a^2}\Longrightarrow q^2-p^2=-a^2\Longrightarrow qdq-pdp=0$$

$$\Longrightarrow d(\ln(p+q))=\frac{d(p+q)}{p+q}=\frac{dq}{p}=\frac{dp}{q}=\frac{pdq}{p^2}=\frac{qdp}{q^2}=\frac{qdp-pdq}{q^2-p^2}=-\frac{qdp-pdq}{a^2}$$

$$\int\frac{\sqrt{x^2-a^2}}{x^2}dx=\int\frac{q}{p^2}dp=-\int qd\left(\frac{1}{p}\right)=-\frac{q}{p}+\int\frac{dq}{p}=-\frac{q}{p}+\int\frac{d(p+q)}{p+q}$$

$$=-\frac{q}{p}+\int d(\ln(p+q))=-\frac{q}{p}+\ln(p+q)+C=-\frac{\sqrt{x^2-a^2}}{x}+\ln\left(x+\sqrt{x^2-a^2}\right)+C$$

$$5.(1)\int(e^x-1-x)^2dx=\int(e^{2x}+1+x^2+2x-2xe^x-2e^x)dx=\frac{1}{2}e^{2x}+x+\frac{x^3}{3}+x^2-\int 2xe^xdx-2e^x+C$$

$$=\frac{1}{2}e^{2x}+x+\frac{x^3}{3}+x^2-2\int xde^x-2e^x+C=\frac{1}{2}e^{2x}+x+\frac{x^3}{3}+x^2-2\left(xe^x-\int e^xdx\right)-2e^x+C$$

$$=\frac{1}{2}e^{2x}+x+\frac{x^3}{3}+x^2-2(xe^x-e^x)-2e^x+C=\frac{1}{2}e^{2x}+x+\frac{x^3}{3}+x^2-2xe^x+C$$

$$5.(4)\int x\sin^2xdx=\int x\frac{1-\cos 2x}{2}dx=\frac{1}{2}\int xdx-\frac{1}{2}\int x\cos 2xdx=\frac{1}{4}x^2-\frac{1}{4}\int xd\sin 2x+C$$

$$=\frac{1}{4}x^2-\frac{1}{4}\left(x\sin 2x-\int\sin 2xdx\right)+C=\frac{1}{4}x^2-\frac{1}{4}\left(x\sin 2x-\frac{1}{2}\int\sin 2xd2x\right)+C$$

$$=\frac{1}{4}x^2-\frac{1}{4}\left(x\sin 2x+\frac{1}{2}\cos 2x\right)+C=\frac{1}{4}x^2-\frac{1}{4}x\sin 2x-\frac{1}{8}\cos 2x+C$$

$$5.(6)\int x^2\sin^2x\cos^2xdx=\frac{1}{4}\int x^2\sin^2(2x)dx=\frac{1}{4}\int x^2\frac{1-\cos(4x)}{2}dx=\frac{1}{8}\int x^2dx-\frac{1}{8}\int x^2\cos(4x)dx$$

$$=\frac{x^3}{24}-\frac{1}{32}\int x^2d\sin(4x)+C=\frac{x^3}{24}-\frac{1}{32}\left(x^2\sin(4x)-\int\sin(4x)dx^2\right)+C$$

$$=\frac{x^3}{24}-\frac{1}{32}x^2\sin(4x)+\frac{1}{16}\int\sin(4x)xdx+C=\frac{x^3}{24}-\frac{1}{32}x^2\sin(4x)-\frac{1}{64}\int xd\cos(4x)+C$$

$$=\frac{x^3}{24}-\frac{1}{32}x^2\sin(4x)-\frac{1}{64}\left(x\cos(4x)-\int\cos(4x)dx\right)+C$$

$$=\frac{x^3}{24}-\frac{1}{32}x^2\sin(4x)-\frac{1}{64}x\cos(4x)+\frac{1}{256}\sin(4x)+C$$

$$5.(8)\int x^2\arcsin xdx=\frac{1}{3}\int\arcsin xdx^3=\frac{1}{3}\left(x^3\arcsin x-\int x^3d\arcsin x\right)$$

$$=\frac{1}{3}\left(x^3\arcsin x-\int\frac{x^3}{\sqrt{1-x^2}}dx\right)=\frac{1}{3}x^3\arcsin x-\frac{1}{6}\int\frac{x^2}{\sqrt{1-x^2}}dx^2$$

$$=\frac{1}{3}x^3\arcsin x+\frac{1}{3}\int x^2d\sqrt{1-x^2}=\frac{1}{3}x^3\arcsin x+\frac{1}{3}\left(x^2\sqrt{1-x^2}-\int\sqrt{1-x^2}dx^2\right)$$

$$=\frac{1}{3}x^3\arcsin x+\frac{1}{3}x^2\sqrt{1-x^2}-\frac{1}{3}\int\sqrt{1-x^2}dx^2=\frac{1}{3}x^3\arcsin x+\frac{1}{3}x^2\sqrt{1-x^2}+\frac{1}{3}\int\sqrt{1-x^2}d(1-x^2)$$

$$=\frac{1}{3}x^3\arcsin x+\frac{1}{3}x^2\sqrt{1-x^2}+\frac{2}{9}\int d(1-x^2)^{\frac{3}{2}}=\frac{1}{3}x^3\arcsin x+\frac{1}{3}x^2\sqrt{1-x^2}+\frac{2}{9}(1-x^2)^{\frac{3}{2}}+C$$

$$=\frac{1}{3}x^3\arcsin x+\frac{1}{3}x^2\sqrt{1-x^2}+\frac{2}{9}(1-x^2)\sqrt{1-x^2}+C=\frac{1}{3}x^3\arcsin x+\frac{3}{9}x^2\sqrt{1-x^2}+\frac{2}{9}(1-x^2)\sqrt{1-x^2}+C$$

$$=\frac{1}{3}x^3\arcsin x+\frac{1}{9}(x^2+2)\sqrt{1-x^2}+C$$

$$5.(10)\int x\ln\left|\frac{1-x}{1+x}\right|dx=\int xd\left(\frac{1}{x-1}-\frac{1}{x+1}\right)=\left(\frac{1}{x-1}-\frac{1}{x+1}\right)x-\int\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx$$

$$=\frac{2x}{x^2-1}-\ln\left|\frac{1-x}{1+x}\right|+C$$

5.(12)这里默认对ln内进行了符号修正，默认进行了常数修正.

$$\int \ln\left(x+\sqrt{1-x^2}\right)dx \overset{x=\sin t}{=} \int \ln(\sin t+\cos t)\,d\sin t = \int \ln\left(\sqrt{2}\sin\left(t+\frac{\pi}{4}\right)\right)\cos t\,dt$$

$$\overset{t=u-\frac{\pi}{4}}{=} \int \ln(\sqrt{2}\sin u)\cos\left(u-\frac{\pi}{4}\right)dt = \int \ln(\sin u)\cos\left(u-\frac{\pi}{4}\right)du - \int \frac{\ln 2}{2}\cos t\,dt$$

$$=\frac{\sqrt{2}}{2}\int \ln(\sin u)\,(\cos u+\sin u)\,du - \frac{\ln 2}{2}\sin t = \frac{\sqrt{2}}{2}\int \ln(\sin u)\,(\cos u+\sin u)\,du - \frac{\ln 2}{2}x$$

$$=\frac{\sqrt{2}}{2}\int \ln(\sin u)\sin u\,du + \frac{\sqrt{2}}{2}\int \ln(\sin u)\cos u\,du - \frac{\ln 2}{2}x$$

$$lemma: \int \ln x\,dx = x\ln x - \int x\,d\ln x = x\ln x - \int dx = x\ln x - x$$

$$①\int \ln(\sin u)\sin u\,du = -\frac{1}{2}\int \ln(1-\cos^2 u)\,d\cos u = -\frac{1}{2}\left(\int \ln(1-\cos u)\,d\cos u + \int \ln(1+\cos u)\,d\cos u\right)$$

$$=-\frac{1}{2}\left(-\int \ln(1-\cos u)\,d(1-\cos u) + \int \ln(1+\cos u)\,d(1+\cos u)\right)$$

$$=-\frac{1}{2}[-(1-\cos u)\ln(1-\cos u)+(1-\cos u)+(1+\cos u)\ln(1+\cos u)-(1+\cos u)]$$

$$=-\frac{1}{2}[-(1-\cos u)\ln(1-\cos u)+(1+\cos u)\ln(1+\cos u)-2\cos u]$$

$$=-\frac{1}{2}\left[\ln\frac{1+\cos u}{1-\cos u}+\cos u\ln(1-\cos^2 u)-2\cos u\right]$$

$$=-\frac{1}{2}\left[\ln\frac{1+\cos u}{1-\cos u}+\cos u\ln(\sin^2 u)-2\cos u\right]$$

$$=-\frac{1}{2}\ln\frac{1+\cos u}{1-\cos u}-\cos u\ln(\sin u)+\cos u$$

$$②\int \ln(\sin u)\cos u\,du = \int \ln(\sin u)\,d\sin u = \sin u\ln(\sin u)-\sin u$$

我们知道$x=\sin t=\sin\left(u-\frac{\pi}{4}\right) \Longrightarrow u=\arcsin x+\frac{\pi}{4}$

$$\int \ln\left(x+\sqrt{1-x^2}\right)dx = \frac{\sqrt{2}}{2}\int \ln(\sin u)\sin u\,du + \frac{\sqrt{2}}{2}\int \ln(\sin u)\cos u\,du - \frac{\ln 2}{2}x$$

$$=\frac{\sqrt{2}}{2}\left[-\frac{1}{2}\ln\frac{1+\cos u}{1-\cos u}-\cos u\ln(\sin u)+\cos u+\sin u\ln(\sin u)-\sin u\right]-\frac{\ln 2}{2}x$$

$$=\frac{\sqrt{2}}{2}\left[-\frac{1}{2}\ln\frac{1+\cos u}{1-\cos u}+(\sin u-\cos u)\ln(\sin u)+\cos u-\sin u\right]-\frac{\ln 2}{2}x$$

$$=\frac{\sqrt{2}}{2}\left[-\frac{1}{2}\ln\frac{1+\cos u}{1-\cos u}+1+\sqrt{2}\sin\left(u-\frac{\pi}{4}\right)\ln(\sin u)-\sqrt{2}\sin\left(u-\frac{\pi}{4}\right)\right]-\frac{\ln 2}{2}x$$

$$=\frac{\sqrt{2}}{2}\left[-\frac{1}{2}\ln\frac{1+\cos\left(\arcsin x+\frac{\pi}{4}\right)}{1-\cos\left(\arcsin x+\frac{\pi}{4}\right)}+1+\sqrt{2}\sin t\ln\left(\sin\left(\arcsin x+\frac{\pi}{4}\right)\right)-\sqrt{2}x\right]-\frac{\ln 2}{2}x$$

$$=\frac{\sqrt{2}}{2}\left[-\frac{1}{2}\ln\frac{1+\cos\left(\arcsin x+\frac{\pi}{4}\right)}{1-\cos\left(\arcsin x+\frac{\pi}{4}\right)}+1+\sqrt{2}x\ln\left(\frac{\sqrt{2}}{2}\left(x+\sqrt{1-x^2}\right)\right)-\sqrt{2}x\right]-\frac{\ln 2}{2}x$$

$$=-x-\frac{\sqrt{2}}{4}\ln\frac{1+\cos\left(\arcsin x+\frac{\pi}{4}\right)}{1-\cos\left(\arcsin x+\frac{\pi}{4}\right)}+x\ln\left(\frac{\sqrt{2}}{2}\left(x+\sqrt{1-x^2}\right)\right)-\frac{\ln 2}{2}x$$

$$=-\frac{\sqrt{2}}{4}\ln\frac{1+\frac{\sqrt{2}}{2}\left(\sqrt{1-x^2}-x\right)}{1-\frac{\sqrt{2}}{2}\left(\sqrt{1-x^2}-x\right)}+x\ln\left(x+\sqrt{1-x^2}\right)-x$$

终于做出来了!!!

Derivative

Step-by-step solution

$$\frac{d}{dx}\left(-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}+\sqrt{1-x^2}-x\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}-\sqrt{1-x^2}+x\right)+x\log\left(x+\sqrt{1-x^2}\right)-x\right)=\log\left(\sqrt{1-x^2}+x\right)$$

$$6.(2)\int xe^{\sqrt{x}}dx\overset{t=\sqrt{x}}{=}\int t^2e^tdt^2=2\int t^3e^tdt=2\int t^3de^t=2\left(t^3e^t-\int e^tdt^3\right)$$

$$=2t^3e^t-6\int e^tt^2dt=2t^3e^t-6\int t^2de^t=2t^3e^t-6\left(t^2e^t-\int e^tdt^2\right)=2t^3e^t-6t^2e^t+6\int e^tdt^2$$

$$=2t^3e^t-6t^2e^t+12\int te^tdt=2t^3e^t-6t^2e^t+12\int tde^t=2t^3e^t-6t^2e^t+12\left(te^t-\int e^tdt\right)$$

$$=2t^3e^t-6t^2e^t+12te^t-12e^t+C=2x^{\frac{3}{2}}e^{\sqrt{x}}-6xe^{\sqrt{x}}+12\sqrt{x}e^{\sqrt{x}}-12e^{\sqrt{x}}+C$$

$$6.(6)\int x\sin\sqrt{x}dx\overset{t=\sqrt{x}}{=}\int t^2\sin tdt^2=\int 2t^3\sin tdt=-\int 2t^3d\cos t=-2t^3\cos t+\int\cos td2t^3$$

$$=-2t^3\cos t+6\int t^2\cos tdt=-2t^3\cos t+6\int t^2d\sin t=-2t^3\cos t+6\left(t^2\sin t-\int\sin tdt^2\right)$$

$$=-2t^3\cos t+6t^2\sin t-12\int t\sin tdt=-2t^3\cos t+6t^2\sin t+12\int td\cos t$$

$$=-2t^3\cos t+6t^2\sin t+12t\cos t-12\int\cos tdt=-2t^3\cos t+6t^2\sin t+12t\cos t-12\sin t+C$$

$$=-2x^{\frac{3}{2}}\cos\sqrt{x}+6x\sin\sqrt{x}+12\sqrt{x}\cos\sqrt{x}-12\sin\sqrt{x}+C$$

$$6.(10)\int\sqrt{1-x^2}e^{\arccos x}dx\overset{x=\cos t}{=}\int\sin te^td\cos t=-\int e^t\sin^2tdt=-\int e^t\frac{1-\cos(2t)}{2}dt$$

$$=-\frac{1}{2}\int e^t(1-\cos(2t))dt=-\frac{1}{2}\int e^tdt+\frac{1}{2}\int e^t\cos(2t)dt=-\frac{1}{2}e^t+C+\frac{1}{2}\int e^t\cos(2t)dt$$

先计算$\int e^t\cos(2t)dt=\int\cos(2t)de^t=e^t\cos(2t)-\int e^td\cos(2t)=e^t\cos(2t)+2\int e^t\sin(2t)dt$

$$=e^t\cos(2t)+2\int\sin(2t)de^t=e^t\cos(2t)+2\left(e^t\sin(2t)-\int e^td\sin(2t)\right)$$

$$=e^t\cos(2t)+2e^t\sin(2t)-4\int e^t\cos(2t)dt=\frac{1}{5}e^t\cos(2t)+\frac{2}{5}e^t\sin(2t)$$

$$\Longrightarrow\int\sqrt{1-x^2}e^{\arccos x}dx=-\frac{1}{2}e^t+C+\frac{1}{2}\int e^t\cos(2t)dt=-\frac{1}{2}e^t+C+\frac{1}{2}\left(\frac{1}{5}e^t\cos(2t)+\frac{2}{5}e^t\sin(2t)\right)$$

$$=-\frac{1}{2}e^t+\frac{1}{10}e^t\cos(2t)+\frac{1}{5}e^t\sin(2t)+C$$

$$=-\frac{1}{2}e^{\arccos x}+\frac{1}{10}e^{\arccos x}\cos(2\arccos x)+\frac{1}{5}e^{\arccos x}\sin(2\arccos x)+C$$

$$=-\frac{1}{2}e^{\arccos x}+\frac{1}{10}e^{\arccos x}(2\cos^2(\arccos x)-1)+\frac{2}{5}e^{\arccos x}\sin(\arccos x)\cos(\arccos x)+C$$

$$=-\frac{1}{2}e^{\arccos x}+\frac{1}{10}e^{\arccos x}(2x^2-1)+\frac{2}{5}e^{\arccos x}x\sqrt{1-x^2}+C$$

$$=-\frac{3}{5}e^{\arccos x}+\frac{1}{5}e^{\arccos x}x^2+\frac{2}{5}e^{\arccos x}x\sqrt{1-x^2}+C$$

$$6.(14)\int\frac{\arctan e^x}{e^x}dx=-\int\arctan e^x de^{-x}\overset{t=e^{-x}}{=}-\int\arctan\frac{1}{t}dt=-t\arctan\frac{1}{t}+\int td\arctan\frac{1}{t}$$

$$=-t\arctan\frac{1}{t}+\int\frac{t\left(-\frac{1}{t^2}\right)}{1+\frac{1}{t^2}}dt=-t\arctan\frac{1}{t}-\int\frac{t}{t^2+1}dt=-t\arctan\frac{1}{t}-\frac{1}{2}\int\frac{1}{t^2+1}dt^2$$

$$=-t\arctan\frac{1}{t}-\frac{1}{2}\ln(t^2+1)+C=-e^{-x}\arctan e^x-\frac{1}{2}\ln(e^{-2x}+1)+C$$

$$6.(18)\int\frac{x\arctan x}{(1+x^2)^2}dx=\left(-\frac{1}{2}\right)\int\arctan x\,d\left(\frac{1}{1+x^2}\right)=\left(-\frac{1}{2}\right)\frac{\arctan x}{1+x^2}+\frac{1}{2}\int\frac{1}{1+x^2}d(\arctan x)$$

$$=-\frac{\arctan x}{2(1+x^2)}+\frac{1}{2}\int\frac{1}{1+x^2}d(\arctan x)=-\frac{\arctan x}{2(1+x^2)}+\frac{1}{2}\int\frac{1}{(1+x^2)^2}dx$$

$$\int\frac{1}{(1+x^2)^2}dx=\int\frac{x^2+1-x^2}{(1+x^2)^2}dx=\int\frac{1}{1+x^2}dx-\int\frac{x^2}{(1+x^2)^2}dx=\arctan x+\frac{1}{2}\int xd\left(\frac{1}{1+x^2}\right)+C$$

$$=\arctan x+\frac{1}{2}\frac{x}{1+x^2}-\frac{1}{2}\int\frac{1}{1+x^2}dx+C=\frac{1}{2}\arctan x+\frac{1}{2}\frac{x}{1+x^2}+C$$

$$\Longrightarrow\int\frac{x\arctan x}{(1+x^2)^2}dx=-\frac{\arctan x}{2(1+x^2)}+\frac{1}{2}\int\frac{1}{(1+x^2)^2}dx=-\frac{\arctan x}{2(1+x^2)}+\frac{1}{2}\left(\frac{1}{2}\arctan x+\frac{1}{2}\frac{x}{1+x^2}+C\right)$$

$$=-\frac{\arctan x}{2(1+x^2)}+\frac{1}{4}\arctan x+\frac{1}{4}\frac{x}{1+x^2}+C.$$

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$$1.(2)\int\frac{xdx}{x^3-3x+2}=\int\frac{xdx}{(x-1)^2(x+2)}=\int\frac{(x-1)dx}{(x-1)^2(x+2)}+\int\frac{dx}{(x-1)^2(x+2)}$$

$$=\int\frac{dx}{(x-1)(x+2)}+\int\frac{dx}{(x-1)^2(x+2)}=\frac{1}{3}\int\left(\frac{dx}{x-1}-\frac{dx}{x+2}\right)+\int\frac{dx}{(x-1)^2(x+2)}$$

$$=\int\frac{(x+2)dx}{(x-1)^2(x+2)}-\int\frac{2dx}{(x-1)^2(x+2)}=\int\frac{dx}{(x-1)^2}-2\int\frac{dx}{(x-1)^2(x+2)}$$

$$=\frac{2}{3}\left(\frac{1}{3}\int\left(\frac{dx}{x-1}-\frac{dx}{x+2}\right)+\int\frac{dx}{(x-1)^2(x+2)}\right)+\frac{1}{3}\left(\int\frac{dx}{(x-1)^2}-2\int\frac{dx}{(x-1)^2(x+2)}\right)$$

$$=\frac{2}{9}\int\left(\frac{dx}{x-1}-\frac{dx}{x+2}\right)+\frac{1}{3}\int\frac{dx}{(x-1)^2}=\frac{2}{9}\ln\left|\frac{x-1}{x+2}\right|-\frac{1}{3}\frac{1}{x-1}+C$$

$$1.(4)\int\frac{dx}{x^3+x^2+x+1}=\int\frac{(x-1)dx}{x^4-1}=\int\frac{xdx}{x^4-1}-\int\frac{dx}{x^4-1}=\frac{1}{2}\int\frac{d(x^2)}{(x^2-1)(x^2+1)}-\int\frac{dx}{(x^2-1)(x^2+1)}$$

$$=\frac{1}{4}\int\left(\frac{d(x^2-1)}{x^2-1}-\frac{d(x^2+1)}{x^2+1}\right)-\frac{1}{2}\left(\int\frac{dx}{x^2-1}-\frac{dx}{x^2+1}\right)$$

$$=\frac{1}{4}\int\left(\frac{d(x^2-1)}{x^2-1}-\frac{d(x^2+1)}{x^2+1}\right)-\frac{1}{2}\left(\int\frac{d(x-1)}{2(x-1)}-\frac{d(x+1)}{2(x+1)}-\frac{dx}{x^2+1}\right)$$

$$=\frac{1}{4}\ln\left|\frac{x^2-1}{x^2+1}\right|-\frac{1}{2}\left(\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|-\arctan x\right)+C=\frac{1}{4}\ln\left|\frac{x^2-1}{x^2+1}\right|-\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|+\frac{1}{2}\arctan x+C$$

$$=\frac{1}{4}\ln\left|\frac{(x+1)^2}{x^2+1}\right|+\frac{1}{2}\arctan x+C$$

$$1.(8)\int\frac{x^2+1}{x^4+x^2+1}dx=\int\frac{1+\frac{1}{x^2}}{x^2+1+\frac{1}{x^2}}dx=\int\frac{1}{\left(x-\frac{1}{x}\right)^2+3}d\left(x-\frac{1}{x}\right)=\frac{\sqrt{3}}{3}\int\frac{1}{\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right)^2+1}d\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right)$$

$$=\frac{\sqrt{3}}{3}\arctan\left(\frac{x-\frac{1}{x}}{\sqrt{3}}\right)+C$$

$$1.(11)\int\frac{xdx}{(x-1)^2(x+1)^3}=\frac{1}{4}\int\frac{((x+1)^2-(x-1)^2)dx}{(x-1)^2(x+1)^3}=\frac{1}{4}\left(\int\frac{dx}{(x-1)^2(x+1)}-\int\frac{d(x+1)}{(x+1)^3}\right)$$

$$=\frac{1}{4}\left(\frac{1}{2}\int\frac{((x+1)-(x-1))dx}{(x-1)^2(x+1)}-\int\frac{d(x+1)}{(x+1)^3}\right)=\frac{1}{4}\left(\frac{1}{2}\int\frac{d(x-1)}{(x-1)^2}-\frac{1}{2}\int\frac{dx}{(x+1)(x-1)}-\int\frac{d(x+1)}{(x+1)^3}\right)$$

$$=\frac{1}{8}\int\frac{d(x-1)}{(x-1)^2}-\frac{1}{16}\int\frac{dx}{x-1}+\frac{1}{16}\int\frac{dx}{x+1}-\frac{1}{4}\int\frac{d(x+1)}{(x+1)^3}$$

$$=-\frac{1}{8}\frac{1}{x-1}+\frac{1}{16}\ln\left|\frac{x+1}{x-1}\right|+\frac{1}{8}\frac{1}{(x-1)^2}+C$$

$$2.(1)\int\frac{dx}{x(x^{10}+2)}=\int\frac{x^9dx}{x^{10}(x^{10}+2)}=\frac{1}{10}\int\frac{dx^{10}}{x^{10}(x^{10}+2)}=\frac{1}{10}\int\frac{dx^{10}}{(x^{10}+1)^2-1}=-\frac{1}{20}\operatorname{arctanh}(x^{10}+1)+C$$

$$2.(4)\int\frac{x^4+1}{x(x^4+5)(x^5+5x+1)^2}dx=\frac{1}{5}\int\frac{d(x^5+5x)}{(x^5+5x)(x^5+5x+1)^2}$$

$$=\frac{1}{5}\int\frac{(x^5+5x+1)-(x^5+5x)}{(x^5+5x)(x^5+5x+1)^2}d(x^5+5x)$$

$$=\frac{1}{5}\int\frac{1}{(x^5+5x)(x^5+5x+1)}d(x^5+5x)-\frac{1}{5}\int\frac{d(x^5+5x+1)}{(x^5+5x+1)^2}$$

$$=\frac{1}{5}\int\left(\frac{1}{x^5+5x}-\frac{1}{x^5+5x+1}\right)d(x^5+5x)-\frac{1}{5}\int\frac{d(x^5+5x+1)}{(x^5+5x+1)^2}$$

$$=\frac{1}{5}\ln\left|\frac{x^5+5x}{x^5+5x+1}\right|+\frac{1}{5}\frac{1}{x^5+5x+1}+C$$

$$2.(8)\int\frac{x^5-x}{x^8+1}dx=\frac{1}{2}\int\frac{1}{x^4+x^{-4}}d(x^2+x^{-2})=\frac{1}{2}\int\frac{1}{(x^2+x^{-2})^2-2}d(x^2+x^{-2})$$

$$=\frac{\sqrt{2}}{4}\int\frac{1}{\left(\frac{x^2+x^{-2}}{\sqrt{2}}\right)^2-1}d\left(\frac{x^2+x^{-2}}{\sqrt{2}}\right)=-\frac{\sqrt{2}}{4}\operatorname{arctanh}\left(\frac{x^2+x^{-2}}{\sqrt{2}}\right)$$

$$2.(10)\int\frac{x^2dx}{x^3+x^2+x+1}=\int\frac{x^2(x-1)dx}{x^4-1}=\frac{1}{4}\int\frac{dx^4}{x^4-1}-\int\left(\frac{1}{x^2-1}+\frac{1}{x^2+1}\right)dx=\frac{1}{4}\ln|x^4-1|+\operatorname{arctanh}x+\arctan x+C$$

$$4.u=\frac{x-a}{x-b}\Longrightarrow x=\frac{a-bu}{1-u}=\frac{a-b}{1-u}+b$$

$$\int\frac{dx}{(x-a)^m(x-b)^n}=\int\frac{d\left(\frac{a-b}{1-u}+b\right)}{u^m(x-b)^{n+m}}=\int\frac{d\left(\frac{a-b}{1-u}\right)}{u^m\left(\frac{a-b}{1-u}\right)^{n+m}}=\left(\frac{1}{a-b}\right)^{m+n-1}\int\frac{(1-u)^{n+m-2}du}{u^m}$$

$$=\left(\frac{1}{a-b}\right)^{m+n-1}\int\frac{\sum_{k=0}^{n+m-2}C_{n+m-2}^{k}(-1)^k u^k du}{u^m}=\left(\frac{1}{a-b}\right)^{m+n-1}\int\sum_{k=0}^{n+m-2}C_{n+m-2}^{k}(-1)^k u^{k-m}du$$

$$=\left(\frac{1}{a-b}\right)^{m+n-1}\sum_{k=0}^{n+m-2}C_{n+m-2}^{k}(-1)^k\int u^{k-m}du=\left(\frac{1}{a-b}\right)^{m+n-1}\sum_{k=0}^{n+m-2}C_{n+m-2}^{k}(-1)^k\frac{u^{k-m+1}}{k-m+1}+C$$

$$=\left(\frac{1}{a-b}\right)^{m+n-1}\sum_{k=0}^{n+m-2}C_{n+m-2}^{k}(-1)^k\frac{\left(\frac{x-a}{x-b}\right)^{k-m+1}}{k-m+1}+C$$