1.不妨以C为原点,以v的方向为第一根坐标轴,

以$\overrightarrow{CB}$为第二根坐标轴,建立空间直角坐标系.c_0表示光速

于是这个三角形三个顶点的世界线为$A(ct,-b,0,0)$

$B(c_0t,0,a,0),C(c_0t,0,0,0)$

于是小明的以速度$(v,0,0)$做匀速直线运动

他的世界线为$(c_0t,vt,0,0)$,即$t(c_0,v,0,0)$

设$(c_0t_1,-b,0,0),(c_0t_2,0,a,0),(c_0t_3,0,0,0)$对于小明来说是同时发生的

则$(c_0t_1,-b,0,0)-(c_0t_2,0,a,0)$，$(c_0t_2,0,a,0)-(c_0t_3,0,0,0)$

$(c_0t_1,-b,0,0)-(c_0t_3,0,0,0)$与$(c_0,v,0,0)$正交

即$(c_0(t_1-t_2),-b,-a,0),(c_0(t_2-t_3),0,a,0),(c_0(t_1-t_3),-b,0,0)$与$(c_0,v,0,0)$正交

于是$\begin{cases} c_0^2(t_1-t_2)-bv=0 \\ c_0^2(t_2-t_3)=0 \\ c_0^2(t_1-t_3)-bv=0 \end{cases} \Rightarrow \begin{cases} t_1-t_2=\dfrac{bv}{c_0^2} \\ t_2=t_3 \\ t_1-t_3=\dfrac{bv}{c_0^2} \end{cases}$

考虑空间坐标,则:$\begin{cases} (C-A,C-A)=c_0^2(t_3-t_1)^2-b^2=-\hat{b}^2 \\ (B-C,B-C)=c_0^2(t_2-t_3)^2-a^2=-\hat{a}^2 \\ (A-B,A-B)=c_0^2(t_2-t_1)^2-c^2=-\hat{c}^2 \end{cases}$

于是$\begin{cases} c_0^2\left(\dfrac{bv}{c_0^2}\right)^2-b^2=-\hat{b}^2 \\ -a^2=-\hat{a}^2 \\ c_0^2\left(\dfrac{bv}{c_0^2}\right)^2-c^2=-\hat{c}^2 \end{cases} \Rightarrow \begin{cases} \hat{a}=a \\ \hat{b}=\sqrt{1-\dfrac{v^2}{c_0^2}}\,b \\ \hat{c}^2=c^2-\dfrac{b^2v^2}{c_0^2} \end{cases} \Rightarrow \begin{cases} a'=a \\ b'=\sqrt{1-\dfrac{v^2}{c_0^2}}\,b \\ c'=\sqrt{c^2-\dfrac{b^2v^2}{c_0^2}} \end{cases} \Rightarrow \hat{a}^2+\hat{b}^2=\hat{c}^2.$

$$\tan\theta'=\frac{\hat{a}}{\hat{b}}=\frac{a}{\sqrt{1-\frac{v^2}{c_0^2}}\,b}=\frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}}\frac{a}{b}=\frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}}\tan\theta.\square$$

2. $|(\alpha,\beta)| \geq |\alpha|\cdot|\beta|$

$\Leftrightarrow |x_0y_0 - x_1y_1| \geq \sqrt{x_0^2 - x_1^2}\cdot\sqrt{y_0^2 - y_1^2}$

$\Leftrightarrow |x_0y_0 - x_1y_1|^2 \geq (x_0^2 - x_1^2)\cdot(y_0^2 - y_1^2)$

$\Leftrightarrow x_0^2y_0^2 - 2x_0y_0x_1y_1 + x_1^2y_1^2 \geq x_0^2y_0^2 + x_1^2y_1^2 - x_1^2y_0^2 - x_0^2y_1^2$

$\Leftrightarrow x_1^2y_0^2 + x_0^2y_1^2 \geq 2x_0y_0x_1y_1$

$\Leftrightarrow (x_1y_0 - x_0y_1)^2 \geq 0$(显然)

等号成立当且仅当$x_1y_0 = x_0y_1$,即α,β线性相关.□

3. $(\alpha+\beta,\alpha+\beta) = (x_0+y_0)^2 - (x_1+y_1)^2 = (\alpha,\alpha) + (\beta,\beta) + 2(\alpha,\beta) \geq 0$

$|\alpha+\beta| \geq |\alpha| + |\beta| \Leftrightarrow (\alpha+\beta,\alpha+\beta) \geq (|\alpha|+|\beta|)^2 = (\alpha,\alpha) + (\beta,\beta) + 2|\alpha|\cdot|\beta|$

$\Leftrightarrow (\alpha,\alpha) + (\beta,\beta) + 2(\alpha,\beta) \geq (\alpha,\alpha) + (\beta,\beta) + 2|\alpha|\cdot|\beta|$

$\Leftrightarrow (\alpha,\beta) \geq |\alpha|\cdot|\beta|$

$\overset{(\alpha,\beta)\geq 0}{\Leftrightarrow} |(\alpha,\beta)| \geq |\alpha|\cdot|\beta|$ 由2.知显然.□