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$$1.(1)\begin{bmatrix}0&1&1&-1&2\\0&2&-2&-2&0\\0&-1&-1&1&1\\1&1&0&1&-1\end{bmatrix}\sim\begin{bmatrix}1&1&0&1&-1\\0&1&1&-1&2\\0&-1&-1&1&1\\0&2&-2&-2&0\end{bmatrix}\sim\begin{bmatrix}1&1&0&1&-1\\0&1&1&-1&2\\0&0&0&0&3\\0&0&-4&0&-4\end{bmatrix}\sim\begin{bmatrix}1&1&0&1&-1\\0&1&1&-1&2\\0&0&-4&0&-4\\0&0&0&0&3\end{bmatrix}$$

$$\Rightarrow rank\left(\begin{bmatrix}0&1&1&-1&2\\0&2&-2&-2&0\\0&-1&-1&1&1\\1&1&0&1&-1\end{bmatrix}\right)=4$$

$$1.(3)\begin{bmatrix}14&12&6&8&2\\6&104&21&9&17\\7&6&3&4&1\\35&30&15&20&5\end{bmatrix}\sim\begin{bmatrix}7&6&3&4&1\\6&104&21&9&17\\7&6&3&4&1\\7&6&3&4&1\end{bmatrix}\sim\begin{bmatrix}7&6&3&4&1\\6&104&21&9&17\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$$

$$\sim\begin{bmatrix}42&36&18&24&6\\42&728&147&63&119\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\sim\begin{bmatrix}42&36&18&24&6\\0&-692&-129&-39&-113\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$$

$$\Rightarrow rank\left(\begin{bmatrix}14&12&6&8&2\\6&104&21&9&17\\7&6&3&4&1\\35&30&15&20&5\end{bmatrix}\right)=2$$

$$2.(1)\begin{bmatrix}\alpha_1&\alpha_2&\alpha_3&\alpha_4\end{bmatrix}=\begin{bmatrix}6&1&1&7\\4&0&4&1\\1&2&-9&0\\-1&3&-16&-1\\2&-4&22&3\end{bmatrix}\sim\begin{bmatrix}1&2&-9&0\\4&0&4&1\\6&1&1&7\\-1&3&-16&-1\\2&-4&22&3\end{bmatrix}\sim\begin{bmatrix}1&2&-9&0\\0&-8&40&1\\0&-11&55&7\\0&5&-25&-1\\0&-8&40&3\end{bmatrix}\sim\begin{bmatrix}1&2&-9&0\\0&1&-5&-\frac{1}{8}\\0&0&0&-\frac{45}{88}\\0&0&0&-\frac{3}{40}\\0&0&0&2\end{bmatrix}$$

$$\Rightarrow rank\left(\begin{bmatrix}\alpha_1&\alpha_2&\alpha_3&\alpha_4\end{bmatrix}\right)=3$$

$(\alpha_1,\alpha_2,\alpha_4)$是其极大线性无关部分组

$$3.A=\begin{bmatrix}1 & \lambda & -1 & 2\\ 2 & -1 & \lambda & 5\\ 1 & 10 & -6 & 1\end{bmatrix}\sim\begin{bmatrix}1 & \lambda & -1 & 2\\ 0 & -2\lambda-1 & \lambda+2 & 1\\ 0 & -\lambda+10 & -5 & -1\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & \lambda & -1 & 2\\ 0 & -21 & \lambda+12 & 3\\ 0 & -\lambda+10 & -5 & -1\end{bmatrix}$$

$if\ \lambda=10, rank(A)=2,$

$if\ \lambda\neq 10, rank(A)=3.$

10.法一：

$$r(A_{m\times n})+r(B_{m\times n})=r\begin{pmatrix}A_{m\times n} & 0_{m\times n}\\ 0_{m\times n} & B_{m\times n}\end{pmatrix}=r\begin{pmatrix}A_{m\times n} & B_{m\times n}\\ 0_{m\times n} & B_{m\times n}\end{pmatrix}=r\begin{pmatrix}A_{m\times n}+B_{m\times n} & B_{m\times n}\\ B_{m\times n} & B_{m\times n}\end{pmatrix}$$

$$=r\begin{pmatrix}A_{m\times n}+B_{m\times n} & -A_{m\times n}\\ B_{m\times n} & 0\end{pmatrix}\geq r\begin{pmatrix}A_{m\times n}+B_{m\times n}\\ B_{m\times n}\end{pmatrix}\geq r(A_{m\times n}+B_{m\times n})=r(C)$$

法二：

$denote\ A\ by\ (\text{col1}\quad \text{col2}\quad \cdots\quad \text{coln}),$

$denote\ B\ by\ (\text{col1'}\quad \text{col2'}\quad \cdots\quad \text{coln'}),$

$thus\ C=A+B=(\text{col1+col1'}\quad \text{col2+col2'}\quad \cdots\quad \text{coln+coln'}).$

$\bullet\text{if } \forall i\in\{1,2,\cdots n\}, (\text{col}i, \text{col1'}, \text{col2'}, \cdots, \text{coln'})\ independent$

$then\ r(C)=r(A)+r(B)$

$\bullet\text{if } \exists i\in\{1,2,\cdots n\}, (\text{col}i, \text{col1'}, \text{col2'}, \cdots, \text{coln'})\ dependent$

$then\ r(C)<r(A)+r(B)$

$Hence, r(C)\leq r(A)+r(B)$

13.since $r(A)=0\ or\ 1,$

we can denote A by $(c_1\alpha\quad c_2\alpha\quad \cdots\quad c_n\alpha), \forall j\in\{1,2,\cdots,n\}, c_j\in K.$

α is a m$\times$1column vector $\Rightarrow \alpha\in K^m$

$A=(c_1\alpha\quad c_2\alpha\quad \cdots\quad c_n\alpha)=\alpha(c_1\quad c_2\quad \cdots\quad c_n)$

$$choose\begin{pmatrix}a_1\\ a_2\\ \vdots\\ a_m\end{pmatrix}=\alpha, (b_1\quad b_2\quad \cdots\quad b_n)=(c_1\quad c_2\quad \cdots\quad c_n),$$

$thus\ \forall i\in\{1,2,\cdots,m\}, a_i\in K, \forall j\in\{1,2,\cdots,n\}, b_j\in K.$

$$then\ A=\alpha(c_1\quad c_2\quad \cdots\quad c_n)=\begin{pmatrix}a_1\\ a_2\\ \vdots\\ a_m\end{pmatrix}(b_1\quad b_2\quad \cdots\quad b_n)=\begin{pmatrix}a_1b_1 & a_1b_2 & \cdots & a_1b_n\\ a_2b_1 & a_2b_2 & \cdots & a_2b_n\\ \vdots & \vdots & & \vdots\\ a_mb_1 & a_mb_2 & \cdots & a_mb_n\end{pmatrix}$$

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$4.proof$:

V显然符合原有向量加法法则

下面证明：V关于新定义的数乘不构成K上的线性空间

$$(1)1\circ\alpha=\frac{1}{1}\alpha=\alpha$$

$$(2)k\circ(l\circ\alpha)=k\circ\frac{\alpha}{l}=\frac{\alpha}{kl}=l\circ\frac{\alpha}{k}=l\circ(k\circ\alpha)$$

$$(3)k\circ(\alpha+\beta)=\frac{\alpha+\beta}{k}=\frac{\alpha}{k}+\frac{\beta}{k}=k\circ\alpha+k\circ\beta$$

$$(4)(k+l)\circ\alpha=\frac{\alpha}{k+l},$$

但$k\circ\alpha+l\circ\alpha=\frac{\alpha}{k}+\frac{\alpha}{l}=\frac{(k+l)\alpha}{kl}$不恒等于$\frac{\alpha}{k+l}=(k+l)\circ\alpha$，不成立

故V关于新定义的数乘不构成K上的线性空间.